Math 421 – Dynamical Systems and Chaos
Preliminary Syllabus

Continuous and discrete time dynamical systems. Examples from mathematics, physics, chemistry, biology, economics, engineering etc. Some history. Conservative and dissipative systems. Relations between continuous and discrete systems: Poincaré maps, discretization and numerical analysis.

General notions in dynamics

One-dimensional dynamics:
• Linear systems and their behaviour.
• Nonlinear systems and the continuous/discrete dichotomy: limited behaviour of continuous time systems vs. rich behaviour of discrete time systems.

Discrete 1d dynamics
Examples: the quadratic family, the doubling map.
Newton’s method as a discrete dynamical system.
Phase portraits and graphical analysis (“cobweb diagrams”).
Toward a mathematical definition of “chaos”.
Sarkovskii’s theorem and “Period 3 implies chaos”.
Cantor sets, symbolic dynamics.
The role of the Schwarzian derivative.
Maps of the circle, rotation number, Morse-Smale maps.
Period-doubling bifurcations, the period-doubling route to chaos and the Feigenbaum number.

Two-dimensional dynamics:

(i) Continuous time systems
Linear systems and their classification.
Nonlinear systems, phase plane analysis, periodic orbits, limit cycles.
The Poincaré-Bendixson Theorem (no “chaos” in 2d continuous systems).
The Hopf bifurcation.

(ii) Discrete systems
The Hénon and horseshoe maps.
Attractors and “strange” attractors.
Complex-valued dynamics:
● The Julia and the filled Julia sets of a complex polynomial.
● The basin of attraction. The quadratic family revisited.
● The Fundamental Dichotomy for Julia sets of quadratic maps.
● The Mandelbrot set and its relation to Julia sets.
● Further properties of the Mandelbrot set.
● The Newton method revisited (for complex functions).

Higher-dimensional dynamics:

Continuous time systems:
Three dimensions is enough for “chaos”, Lorenz system and its attractor.
The three-body problem.
The damped, driven pendulum.

Experimental measures of chaos